1. **Problem**

   Compute the Hessian of the function

   $$f(x_1, x_2) = 7x_1^2 + 5x_1 x_2 + 3x_2^2$$

   at $(x_1, x_2) = (1, 4)$. What is the value of the upper left element?

   (a) 6

   (b) 7

   (c) 14

   (d) 5

   (e) -19

   Solution

   The first-order partial derivatives are

   $$\begin{aligned} f_1'(x_1, x_2) &= 14x_1 + 5x_2 \\ f_2'(x_1, x_2) &= 5x_1 + 6x_2 \end{aligned}$$

   and the second-order partial derivatives are

   $$\begin{aligned} f_{11}''(x_1, x_2) &= 14 \\ f_{12}''(x_1, x_2) &= 5 \\ f_{21}''(x_1, x_2) &= 5 \\ f_{22}''(x_1, x_2) &= 6 \end{aligned}$$

   Therefore the Hessian is

   $$f''(x_1, x_2) = \begin{pmatrix} 14 & 5 \\ 5 & 6 \end{pmatrix}$$

   independent of x_1 and x_2. Thus, the upper left element is: $f_{11}''(1, 4) = 14$.

   (a) False

   (b) False

   (c) True

   (d) False

   (e) False