1. **Problem**
For the matrix

$$A = \begin{pmatrix} 16 & -12 & -12 & -16 \\ -12 & 25 & 1 & -4 \\ -12 & 1 & 17 & 14 \\ -16 & -4 & 14 & 57 \end{pmatrix}.$$

compute the matrix $L = (\ell_{ij})_{1 \le i,j \le 4}$ from the Cholesky decomposition $A = LL^\top$.
Which of the following statements are true?

(a) $\ell_{41} \ge -4$
(b) $\ell_{33} \ge 2$
(c) $\ell_{11} \le 4$
(d) $\ell_{31} \ge -3$
(e) $\ell_{32} \ge -2$

Solution
The decomposition yields

$$L = \begin{pmatrix} 4 & 0 & 0 & 0 \\ -3 & 4 & 0 & 0 \\ -3 & -2 & 2 & 0 \\ -4 & -4 & -3 & 4 \end{pmatrix}$$

and hence:

(a) True. $\ell_{41} = -4$
(b) True. $\ell_{33} = 2$
(c) True. $\ell_{11} = 4$
(d) True. $\ell_{31} = -3$
(e) True. $\ell_{32} = -2$