

1. **Problem**

Compute the Hessian of the function

$$f(x_1, x_2) = -7x_1^2 - 5x_1x_2 - 4x_2^2$$

at $(x_1, x_2) = (-2, 2)$. What is the value of the upper left element?

- (a) -8
- (b) -5
- (c) -16
- (d) -14
- (e) -4

Solution

The first-order partial derivatives are

$$\begin{aligned}f'_1(x_1, x_2) &= -14x_1 - 5x_2 \\f'_2(x_1, x_2) &= -5x_1 - 8x_2\end{aligned}$$

and the second-order partial derivatives are

$$\begin{aligned}f''_{11}(x_1, x_2) &= -14 \\f''_{12}(x_1, x_2) &= -5 \\f''_{21}(x_1, x_2) &= -5 \\f''_{22}(x_1, x_2) &= -8\end{aligned}$$

Therefore the Hessian is

$$f''(x_1, x_2) = \begin{pmatrix} -14 & -5 \\ -5 & -8 \end{pmatrix}$$

independent of x_1 and x_2 . Thus, the upper left element is: $f''_{11}(-2, 2) = -14$.

- (a) False
- (b) False
- (c) False
- (d) True
- (e) False