

1. Problem

Theory: Consider a linear regression of y on x . It is usually estimated with which estimation technique (three-letter abbreviation)?

This estimator yields the best linear unbiased estimator (BLUE) under the assumptions of the Gauss-Markov theorem. Which of the following properties are required for the errors of the linear regression model under these assumptions?

independent / zero expectation / normally distributed / identically distributed / homoscedastic

Application: Using the data provided in `linreg.csv` estimate a linear regression of y on x . What are the estimated parameters?

Intercept:

Slope:

In terms of significance at 5% level:

x and y are not significantly correlated / y increases significantly with x / y decreases significantly with x

Solution

Theory: Linear regression models are typically estimated by ordinary least squares (OLS). The Gauss-Markov theorem establishes certain optimality properties: Namely, if the errors have expectation zero, constant variance (homoscedastic), no autocorrelation and the regressors are exogenous and not linearly dependent, the OLS estimator is the best linear unbiased estimator (BLUE).

Application: The estimated coefficients along with their significances are reported in the summary of the fitted regression model, showing that y increases significantly with x (at 5% level).

Call:

```
lm(formula = y ~ x, data = d)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-0.50503 -0.17149 -0.01047  0.13726  0.69840
```

Coefficients:

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.005094   0.023993  -0.212   0.832
x             0.558063   0.044927  12.421 <2e-16 ***
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 0.2399 on 98 degrees of freedom

Multiple R-squared: 0.6116, Adjusted R-squared: 0.6076

F-statistic: 154.3 on 1 and 98 DF, p-value: < 2.2e-16

Code: The analysis can be replicated in R using the following code.

```
## data
d <- read.csv("linreg.csv")
## regression
m <- lm(y ~ x, data = d)
summary(m)
## visualization
```

```
plot(y ~ x, data = d)
abline(m)
```